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The inefficiency of the immigration-based demographic equilibrium

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Abstract. *This study investigates the impact that a possible increase in fertility rates and the control of immigration have on population stationarity following the approach used in Angrisani and Di Palo (2016). Our paper shows that using immigration for population stationarity leads to a gross inefficiency in terms of size and age distribution of the stationary population. Indeed, we show that immigration-based population stationarity is inefficient as the stationary population results in having a larger size than that obtained bringing the fertility rates to the whole natural level of equilibrium. Furthermore, the immigration-based stationary population results in being unbalanced in terms of ratios between old and young age groups pro the former ones.*

In order to perform population evolution, the Leslie model has been used in a two sex form already applied in a previous work, and in order to introduce the effect of possible changes in fertility or immigration in the model the modified approach of Angrisani and Di Palo (2016), has been used in a slightly simplified version. Numerical illustration is obtained on data by gender for the Italian population.

Keywords: population stationarity, Leslie population model, immigration.

JEL Codes: C02, J10, J11

1. Introduction

It goes without saying that over the last few decades demographic instability of populations is a real issue especially in developed countries. This phenomenon has occurred due principally to the concomitant presence of three phenomena: a drastic reduction in fertility rates, a progressive increase in life expectancies (particularly at old ages), and the baby boom after the Second World War.

The problem of population stabilization is a matter of the greatest importance under the practical profile in many social and economic fields, first of all in the economic area relating to the pension system sustainability. Indeed, all the mandatory pension systems are substantially pay-as-you-go financed, namely the current pension expenditure is paid by the current contributions. Hence, having a workers to pensioners ratio that is substantially stable over time is a key point in order to the pension system sustainability as this ratio gives, on average, the number of contributors available to pay one pension. In addition, also the intergenerational equity, which is has to be a not eluding principle of pension system management, is largely based on the stability of the workers to pensioners ratio. More generally speaking, the problem of population stabilization is extremely in all fields where benefits are financed, in terms of current expenditure, by the active part of a population in favor of its older part, for example, in the health care as well as the welfare

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context. It is particularly important the problem of population stationarity, i.e. the stabilization of the size by age groups as well as of the ratios between age groups. In this paper, we principally focus on this topic.

In order to face up to and solve, in tendential terms, the problem of population stabilization, two tools are generally considered: an increase in the fertility rates up to the natural level of balance, and the immigration use. In our work, the longevity issue, although relevant, will not be taken into account as the primary goal is to verify the efficiency of the two above-mentioned tools. We will jointly use both tools, increasing fertility and immigration, and we will represent their use level by means of specific parameters as in the approach of Angrisani and Di Palo (2016). Specifically, we will numerically illustrate our approach based on Italian data, and in our simulation we will produce fourteen pairs of these parameters, increasingly ordered as the total fertility increases, which lead to population stationarity. For each pair of these parameters, we will provide the tendential size and the old-age dependency ratio of the stationary population. Our simulation results show that both the population size and the old-age dependency ratio are increasing as immigration increases, denoting the inefficiency of the immigration tool in order to the population stationarity.

This paper is structured as follows. Firstly, in Section 2 we briefly review the basics of the Leslie population model. Section 3 illustrates the modified approach to the Leslie model in order to take into account changes in fertility and immigration, following Angrisani and Di Palo (2016). In Section 4, we deal with numerical simulation on Italian data. Lastly, our main conclusions are given in Section 5.

2. The basic population model

In this section, we deal with the well-known population projection model of Leslie see Leslie (1945) and Leslie (1948). Specifically, we refer to the different approach that considers the population divided into the two sexes, as developed in earlier works Angrisani, Attias, Bianchi and Varga (2004), Angrisani, Attias, Bianchi and Varga (2012) and Angrisani, Di Palo, Fantaccione and Palazzo (2013).

As in the classical approach of the Leslie model, age-specific survival and fertility rates are used. Both time and age are discrete variables defined on yearly bases.

Let $N-1$ be the maximum attainable age by individuals being alive whatever their sex, namely no individual is alive at age N .

We consider the population separated in the two sex and grouped in N age classes, for $i=0,1,2,\dots,N-1$, and throughout the paper we denote by:

- $x^F(t)$ and $x^M(t)$ the N-dimensional vectors for the female and male population at time t, respectively;
- α_i the reproduction rate for females of age group i (counting all new-borns of both sexes), with $\alpha_i \geq 0$ for $i=0,1,2,\dots,N-1$;
- $\varphi = \dfrac{x_0^F(0)}{x_0(0)}$ the sex female ratio at birth in year 0;
- α_i^F the female per capita birth rate with $\alpha_i^F = \alpha_i \varphi$ for $i=15,16,\dots,50$ and $\alpha_i^F = 0$ for the remaining age groups;
- ω_i^F and ω_i^M the probabilities of surviving from age i to age $i+1$ for females and males, respectively.

Then, the corresponding $N \times N$ Leslie matrix for the female population is given by

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$$L^F = \begin{bmatrix} 0 & 0 & \dots & \alpha_{15}^F & \dots & \alpha_{50}^F & 0 & \dots & 0 & 0 \\ \omega_0^F & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & \omega_1^F & \dots & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ . & . & \dots & . & \dots & . & . & \dots & . & . \\ 0 & 0 & \dots & 0 & \dots & 0 & 0 & 0 & \omega_{N-2}^F & 0 \end{bmatrix}$$

and the dynamics of the female population is

$$x^F(t+1) = L^F x^F(t) \quad \text{with} \quad t = 0,1,2,\dots$$

Analogously, the $N \times N$ Leslie matrix and the dynamics of the male population are given by

$$L^M = \begin{bmatrix} 0 & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ \omega_0^M & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & \omega_1^M & \dots & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ . & . & \dots & . & \dots & . & . & \dots & . & . \\ 0 & 0 & \dots & 0 & \dots & 0 & 0 & 0 & \omega_{N-2}^M & 0 \end{bmatrix}$$

$$x^M(t+1) = L^M x^M(t) + \frac{(1-\varphi)}{\varphi} e_1 \circ L^F x^F(t) \quad \text{with} \quad t = 0,1,2,\dots$$

where e_1 is the N-dimensional basic vector and $\circ$ denotes the Hadamard product of vectors, $a \circ b = [a_1 b_1, a_2 b_2, \dots, a_n b_n]$.

Expressing the total population by state vector $x = \left[x^F x^M\right]^T$, we have the following dynamics

$$x(t+1) = \Lambda x(t) \quad \text{with} \quad t = 0,1,2,\dots$$

where $2N \times 2N$ matrix Λ is defined as

$$\Lambda = \begin{bmatrix} L^F & 0 \\ d & 0 \\ 0 & L_1^M \end{bmatrix}$$

in which N-dimensional vector d and $(N-1 \times N)$ matrix L_1^M are given by

$$d = \begin{bmatrix} 0 & \dots & 0 & \frac{1-\varphi}{\varphi}\alpha_{15}^F & \dots & \frac{1-\varphi}{\varphi}\alpha_{50}^F & 0 & \dots & 0 \end{bmatrix}$$

$$L_1^M = \begin{bmatrix} \omega_0^M & 0 & \dots & 0 & 0 \\ 0 & \omega_1^M & \dots & 0 & 0 \\ . & . & \dots & . & . \\ 0 & 0 & \dots & \omega_{N-2}^M & 0 \end{bmatrix}$$

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Matrix Λ does not satisfy the conditions of the Perron- Frobenius theorem: it has the N^{th} column zeroed and, hence, it is not irreducible. However, the sub-matrix of the Leslie female matrix, denoted by L_{50}^{F} and obtained considering the first 51 rows and columns of L^F, is proved to be irreducible and primitive, see Angrisani, Attias, Bianchi and Varga (2004). Hence, by means of the application of the Perron-Frobenius theorem, L_{50}^{F} has a unique positive eigenvalue, λ_0, strictly dominant, to which a positive eigenvector, ${}^{50}x^{0F}$ (it is the state vector of the female population aged 0-50), is associated.

As in Angrisani, Attias, Bianchi and Varga (2004), firstly it is proved that here exists a number $s > 0$ such that

$$\lim_{t \to +\infty} \frac{{}^{50}x^{F}(t)}{\lambda_0^t} = s\left({}^{50}x^{0F}\right)$$

and, secondly, it is proved that

$$\lim_{t \to +\infty} \frac{x^{F}(t)}{\lambda_0^t} = s\left(x^{0F}\right),$$

where x^{0F} is a nonnegative eigenvector of matrix L^F associated with positive eigenvalue λ_0, whose first 51 entries are the same of ${}^{50}x^{0F}$. Hence, the existence of the asymptotic behavior for female population is proved.

The same is proved for the male population, i.e.

$$\lim_{t \to +\infty} \frac{x^{M}(t)}{\lambda_0^t} = s\left(x^{0M}\right),$$

where x^{0M} is a nonnegative eigenvector of matrix L^M associated with positive eigenvalue λ_0.

In this way, the existence of a demographic equilibrium for the total population is proved (see Theorem 4 in Angrisani, Attias, Bianchi and Varga (2004), p.63), namely it is proved that: a) there exists a population state, denoted by x^0, which is a nonnegative eigenvector of matrix Λ, associated with positive eigenvalue λ_0; and b), on the long term, the age distribution of the population tends to the equilibrium age distribution in the sense that, for any initial state $x(0)$, there exists $s > 0$ such that

$$\lim_{t \to +\infty} \frac{x(t)}{\lambda_0^t} = s\left(x^{0}\right).$$

3. The Leslie modified model with immigration

Building upon the approach in Angrisani and .Di Palo (2016), we modify the Leslie model reviewed in Section 2 introducing two specific parameters, throughout the paper denoted by d and g, respectively, in order to take into account a possible increase in the total fertility rate and a change in immigration.

In this section, we consider the Leslie matrix for the female population truncated for ages between 0 and 54, which has the female per capita birth rates, α_i^F, on the first row and the probabilities of surviving from age i to age $i+1$, ω_i^F, on the first subdiagonal.

In order to manage the increase in the total fertility rate and a change in immigration levels, we proceed as described in the following.

Referring to the female per capita birth rates, for $i = 10, \ldots, 54$ we have

$${}^{m}\alpha_i^F = \alpha_i^F \left(1+\delta\right)$$

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where $(1+ d)$ is an adjustment factor homogeneous for all ages.

Referring to the female survival rates, placed on the other rows of the truncated female Leslie matrix, for $i = 10,\dots,53$ we have

$$^{m}\omega_i^F = \omega_i^F\,(1+\gamma)$$

where $(1+ g)$ is an adjustment factor homogeneously for all ages. The adjustment factor, $(1+ g)$, is considered in order to include the effect of the immigration for women for ages between 0 and 54. In this way, the female population aged i in year t moves to age $i+1$ in year $t+1$ by the effect of both the survival probabilities (the size decreases) and the considered factor, $(1+ g)$, that represents the change in the female population aged i due to immigration. Hence, denoted the modified Leslie matrix for the female population aged 0-54 by

$$L_{54}^F(\delta,\gamma) = \begin{bmatrix} 0 & 0 & \dots & \alpha_{10}^F(1+\delta) & \dots & \alpha_{53}^F(1+\delta) & \alpha_{54}^F(1+\delta) \\ \omega_0^F\,(1+\gamma) & 0 & \dots & 0 & \dots & 0 & 0 \\ 0 & \omega_1^F\,(1+\gamma) & \dots & 0 & \dots & 0 & 0 \\ . & . & \dots & . & \dots & . & . \\ 0 & 0 & \dots & 0 & \dots & \omega_{53}^F(1+\gamma) & 0 \end{bmatrix},$$

the dynamics of the female population is given by

$$x^F(t+1) = L_{54}^F(\delta,\gamma)x^F(t) \quad \text{with} \quad t = 0,1,2,\dots$$

The female Leslie matrix $L_{54}^F(\delta,\gamma)$, truncated and modified by means of d and g, preserves the properties of irreducibility and primitivity, so it admits a positive, strictly dominant, and simple eigenvalue.

As far as concerned with male population, we analogously consider the truncated Leslie matrix, whose dimension is $55\acute{}\ 55$, and in order to include the immigration effect, we multiply all the rows, with the only exception of the first one, by the same adjustment factor, $(1+\delta)$, homogeneous for all age groups.

For both matrices relating to female and male populations, the same considerations hold in terms of convergence.

In order to the population stationarity, we follow the procedure explained hereinafter. With reference to the modified female Leslie matrix, the d value is determined so that the total fertility rate could increase starting from its initial value up to a value closed to the replacement level, with a fixed increasing step. For each considered value of the total fertility rate, we determine the corresponding adjustment factor, $(1+\delta)$, of increase in the total fertility rate. Finally, the value of the adjustment factor of the survival probabilities is determined in order to make the dominant eigenvalue, λ_0, of the truncated female Leslie matrix equal to 1.

Summing up, by means of adjustment rates d and g, we take into account the effect combined of fertility and immigration in the truncated female Leslie matrix in order to have $\lambda_0 = 1$.

4. Numerical simulation on Italian data

In this section, we present the numerical results of applying the modified Leslie model, in the form above-reviewed, on Italian data.

The procedure described in the previous section is used to forecast the dynamics of the Italian population starting from year 2008. The initial population distribution is provided by Istat, the Italian National Institute of Statistics - is relative to the year 2008. For the survival probabilities values, we rely on data for the Italian population for year 2008 available from the Human Mortality Database (HMD), (see [6] - data downloaded on July 2013), whereas for the fertility rates values, we use data for year 2008 provided by the Eurostat Database (see [5] - data downloaded on July 2013).

The results of our own calculations are reported in Table 1, which has to be read by rows.

Row number	Total Fertlity Rate	Adjustment factor of the Total Fertility Rate (1+δ)	Adjustment factor of the Survival Probability (1+γ)	Female Population aged 0-54 with adjusted immigration Year 2009	Female Population aged 0-54 without adjusted immigration Year 2009	Female Immigrants aged 0-54 Year 2009	Male Population aged 0-54 with immigration Year 2009	Male Population aged 0-54 without immigration Year 2009	Male Immigrants aged 0-54 Year 2009	Total Immigrants aged 0-54 Year 2009	Total Population Year 2400	Old-Age Dependency Ratio (in percent) Year 2400	Yearly Change in Total Population Size Year 2400
0	1,415888	1,000000	1,012523	20166727	19920792	245935	20624133	20372347	251786	497721	68263066	46	______
1	1,500000	1,059406	1,010636	20146394	19937513	208881	20603985	20390476	213509	422390	65283937	44	2979128
2	1,550000	1,094720	1,009564	20135280	19947452	187828	20593002	20401014	191988	379817	63684573	44	1599364
3	1,600000	1,130033	1,008526	20124835	19957392	167443	20582704	20411552	171152	338595	62197492	43	1487081
4	1,650000	1,165347	1,007520	20115018	19967332	147686	20573046	20422089	150957	298643	60811687	42	1385805
5	1,700000	1,200660	1,006544	20105790	19977272	128518	20563992	20432627	131365	259883	59517566	42	1294121
6	1,750000	1,235974	1,005596	20097119	19987211	109908	20555507	20443165	112342	222250	58307785	41	1209782
7	1,800000	1,271287	1,004675	20088971	19997151	91820	20547557	20453703	93854	185674	57172643	41	1135142
8	1,850000	1,306601	1,003780	20081320	20007091	74229	20540114	20464240	75874	150103	56107793	40	1064850
9	1,900000	1,341915	1,002908	20074137	20017031	57106	20533150	20474778	58372	115478	55105659	39	1002133
10	1,950000	1,377228	1,002059	20067400	20026970	40430	20526641	20485316	41325	81755	54162346	39	943313
11	2,000000	1,412542	1,001231	20061085	20036910	24175	20520564	20495853	24711	48886	53273334	38	889013
12	2,050000	1,447855	1,000424	20055172	20046851	8321	20514897	20506391	8506	16827	52433685	38	839648
13	2,076740	1,466741	1,000000	20052166	20052166	0	20512027	20512027	0	0	52002831	38	430854

Table. 1: Results of our own calculation on Italian data for male population.

As an example, we analyze the second row. In the second cell, its is reported the 2008 value of the total fertility rate. In the third cell, the total fertility rate value increased up to 1.5000 is reported. Hence, the adjustment factor, $(1+\delta)$, is determined so that the total fertility rate should have moved form the initial 2008 value (1.4158) to 1.5000; this value is indicated in the fourth cell. This means that the total fertility rate needs to be multiplied by about 1.0594, i.e. to be increased of about 6% in order to raise its value from 1.4158 to 1.5000. In the fifth cell, the adjustment factor of survival probabilities, $(1+\gamma)$, is reported; it is determined so that the female population aged 0 – 54 results in being stable, namely so that the dominant eigenvalue of the truncated female Leslie matrix equals one.

In the sixth and seventh cells, there are reported the values of female population aged 0 – 54 in year 2009 obtained, respectively, with or without adjusting the survival probabilities by immigration effect. The difference between the two values provides the number of female immigrants aged 0 – 54 in year 2009, and this value is shown in the eighth cell of Table 1. In the following three cells, the same analogous values for male population aged 0 – 54 in year 2009 are reported. Finally, in the twelfth cell, the immigration whole effect for the age group $[0,54]$ in year 2009 is shown, this value being equal to the sum of female and male immigrants aged 0 – 54. In the thirteenth cell, the size of the total stationary population is shown, this value referring to the projection for ending year 2400 (it is worth noting, however, that the population stabilizes already after sixty years).

In the fourteenth cell, the old age dependency ratio, defined as the ratio between the population aged 65 and over and the population aged 15 – 64, is evaluated for year 2400. In the last cell, for year 2400, there is

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evaluated the difference between the stationary populations obtained as results of two consecutive different adjustments of both the total fertility rate and the survival probabilities.

5. Conclusions

The present study investigates the impact that an increase in the total fertility rate and a change in immigration level could have on population stationarity.

We use the Leslie population model, in a two sex version already introduced in a previous work, which applies the Perron-Frobenius theory to the sub-matrix of the female population. In this work, following the approach of Angrisani and .Di Palo (2016), we modify the entries of the female population sub-matrix, either those relating to the female per capita birth rates and those relating to the survival probabilities, by means of adjustment factors in order to consider the effect of both an increase in fertility and a change in immigration.

Our paper shows that using immigration for population stationarity leads to a gross inefficiency in terms of both the size and the age distribution of the stationary population. Indeed, the stationary population results in having a greater size than that obtained bringing the fertility rates to the whole natural level of balance. In addition, regarding to the population age distribution, we show that an immigration-based stationary population is unbalanced in terms of ratios between old and younger age groups pro the former ones.

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