A mathematical model for consumers based on aspiration adaptation theory and bounded rationality

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Abstract. This paper describes characteristics of a discrete-time model for a consumers group through bounded rationality theory, where the basis of decision-making is the Aspiration Adaptation Theory (Selten, 1998). Each consumer try to imitate the decision of others related to them based in last time period observation, as long as they are connected by no directed and no weight graph. The models consist of some algorithms for imitation process, income raise through education and the extreme cases. Finally, the paper shows conclusions and extensions.

Keywords: aspiration adaptation theory, bounded rationality, imitation, consumer’s theory.

JEL codes: C63, D03, D71, D81, D85.

1. Introduction

Having unlimited cognitive capacity suggests the ability to process information faster or equal to a computer and all mathematical processes are perfect to make the best decision among a set of infinity time-limitless possibilities. In consumer’s theory it means that the consumer maximizes his utility function restricted to his budget constraint. This perspective is in conventional microeconomic textbooks and it leads to fundamental results in the study of consumer’s behavior.

From a different approach, the consumers behave like human beings and have cognitive limitations. The bounded rationality theory can explain such limitation in an environment. The concept of bounded rationality starts in Simon (1955) who stated the importance of replacing a global rationality in economics for a rational behavioral concept compatible to access information and computational constraints. Later studies like Simon (1957); Simon, Edgi, Viale and Marris (1992); Rubinstein (1998); Gigerenzer & Selten (2002) and Selten (2002) have helped to bounded rational concept. Simon et al. (1992) state the importance of relatives to complex environment, human brain computing capacity limited by time, generating behavioral characteristics suggesting the necessity to build economic models based on human thinking.

The Aspiration Adaptation Theory (AAT) (Sauermann & Selten, 1962; Selten, 1998) offers a different alternative to the optimization process, which goes against bounded rationality theories (Gigerenzer & Selten, 2002). According to AAT, there is no utility or benefit function to quantify consumer’s satisfaction. Instead there are different incomparable goals to define what the economic agent wants. AAT was developed to
explain a unique firm behavior, but it is extensible to a consumer theory. It is a bounded rational theory example which is non-optimizing but not irrational (Selten, 1998).

There are consumer behavioral models like Özak (2009) based on the optimization process, but in this article, the model supposes no function utility of consumption function to serve to an individual decision making tool. There are some models based on AAT like Rosenfeld & Sarit (2011), but in this case it takes incomparable goals and there are many consumers in a community.

2. The theoretical model

2.1. Definitions and notation

It’s a time – discrete model. The variable \( t \in \mathbb{N} \) designates time. There is a finite set of consumers denoted by \( X \), and each consumer is denoted by \( x_i \), \( 1 \leq i \leq n \), and they can be ordered with no generality loss. The community of consumers by a no directed and no weight graph \( \mathcal{G} = (X, E) \) is defined, where \( E \subseteq [X]^2 \) represents the relations among consumers. An element \( e \in E, e = \{x_i, x_j\} \) means consumer \( x_i \) is related to consumer \( x_j \), therefore they can share information, it can be understood as friendship between people (it is unrealistic that a person should share information with himself, which implies \( i \neq j \)).

Every consumer has an income \( v \) at every time \( t \), generating the possibility to save and have monetary wealth. Income on the model allows purchasing available goods in the market, and wealth allows reaching those who cannot buy in a certain time period based on his/her budget constraints. Consumption on an individual depends on his/her actual income and his wealth on the last period of time. There is no financial system or government (no indebtedness or tax on the model).

Each consumer can choose among set of \( r \) goods available on the market. Suppose that individual has two kinds of needs. The first related to their physiological needs as a living being, which can be called food or alimentary good. The second related to their emotional well-being to which we can call non food or non alimentary good. Each good satisfies one or both needs, and has a price that cannot be modified by a consumer.

Since there are \( r \) goods, and every single one with three characteristics, the price in the market and levels that each good contributes to the satisfaction of its alimentary and no alimentary need, respectively in a period of time, it can build a good matrix \( B = (b_{ij}), 1 \leq i \leq r + 1 ; 1 \leq j \leq 3 \), where \( b_{ij} \) corresponds to characteristic \( j \) of good \( i \). These characteristics, for simplicity of the model, are invariant through time and common to all the consumers. Therefore, the contribution of one good for alimentary and no alimentary needs is the same for all. Goods split in turn in \( \alpha \) alimentary goods and \( r - \alpha \) not alimentary goods. Each group must order upwardly to the price.

Additionally the model gives capability of varying consumers’ income by consuming a special kind of good called education, it has an invariant price and can generate an increment \( \delta \) over the income when it is consumed. Education properties are located in the last row of matrix goods, for this reason it has \( r + 1 \) row.

2.2. Instrumental variables and goals

Based on AAT (Selten,1998), each individual has variables that determine his/her goals. The consumer goals correspond to the satisfaction of their needs as well as quest of wealth. Let \( g_1 \) and \( g_2 \) denote correspondent goals to the satisfaction of alimentary and no alimentary needs. The monetary wealth is denoted by \( g_3 \).

\footnote{Following the notation used on Diestel (2000)}
The instrumental variables of every individual $Y_k$, $1 \leq k \leq r + 1$ represent goods that each individual consumes and $y_k$ consumed quantities of each good, taking into account education at position $r + 1$. A plan $y = (y_1, \ldots, y_{r+1})$ is a combination of values for instrumental variables, in this case, represents a number of units consumed by a person in a period of time, this plan varies through time. It will be supposed for simplicity in the model that a person consumes one unit of each good at the most. As each person follows a different plan, the notation to keep is $x_{i,t}$.  

2.3. Actions

An action $y' = A_I(y)$ is a function that indicates the variation in a consumption pattern. Each variation consumption pattern has an effect on the goals. The actions proposed in the model are:

- $A_1$: Imitation on alimentary goods.
- $A_2$: Imitation on non-alimentary goods.
- $A_3$: Increment on person education.
- $A_4$: Saving.

Each of these actions seeks to generate an increment at least in one goal, the last one (reserved for some extreme cases).

2.4. Construction of the influence scheme

The influence scheme corresponds to a $4 \times 3$ matrix, it will denote by $Q$, contains what really happens to a consumer after having consumed goods in certain period of time.

The choice of elements of matrix depends on values of variables $g_k(t)$ and $g_k(t-1)$. When $t = 0$, scheme of influences does not exist, meantime the individual does not begin his process of consumption, it will not be able to evidence variations that generate actions. Therefore the influence scheme must be modified for every $t \geq 1$.

In actions $A_1$ and $A_4$, correspondent to consumption of alimentary goods, it lacks of sense of consumption if it has reached maximum satisfaction level, equivalent to 100 units. Verification for this level of aspiration can be seen in these algorithms. When executing these actions (whichever it had been taken) if $g_1(t) \geq 100$, it should be assigned at the position $q_{41}$ or $q_{44}$ the value of 1, in case that it has not been assigned previously.

It is evident that consumption of alimentary or non-alimentary goods will generate an increase in any one of the variables $g_1$ and $g_2$, however it is the variation $\Delta g_k = g_k(t) - g_k(t - 1)$which will give counts if actions (particularly on imitation in non-alimentary goods) have produced improvement on the goal, and that way, look for a different order in actions that the individual should follow in the next period of time, otherwise, dynamics of model would go away. This is a substantial difference with Selten's original model (Selten, 1998) where influences scheme adds a sign if the goal increases in accordance with a given action, but do not give account of whether you had a positive or negative increment in time.

3. Algorithms for actions and others considerations

3.1. Imitation on consumption of alimentary and non–alimentary goods

In order to generate the imitation in consumption, rule will be: First, you will consume the goods that had bigger consumption inside your social circle in the immediately previous time. This rule contains three processes and they are the following:
1. Build an alimentary (or non-alimentary) goods demand matrix $I$ to their social circle, to every consumer, because every individual have their own social circle and it is different from another (it can be seen on adjacency matrix of graph). The following algorithm allows building a matrix that contains an index on each good on first row, and number of people in social circle who have consumed these goods in the last period of time. Given a consumer $x_i$ it follows this sequence of steps:

For $k$ from 1 to $\alpha$ do:

\[
I_{1k} = k
\]
\[
I_{2k} = 0
\]

For every $x_j$ such that exist $e = \{x_i, x_j\} \in E$ do:

For $k$ from 1 to $\alpha$ do:

If $x_j, y_k(t - 1) \neq 0$

\[
I_{2k} = I_{2k} + 1
\]

2. Sort the columns in demand matrix based in second row downwardly. It is very important that the elements on the first row should change in accordance with changes made on the second row. This allows to know which goods they have consumed more than others. Consumption through observation would implicate that the consumer sees people close to him consuming a good and he will consume it also depending on the people’s number that they had consumed it in a previous time.

3. Consume goods based on his budget constraint and the permutated index in the first row of demand matrix. There are few differences on alimentary and non-alimentary consumption algorithms. For alimentary goods:

Begin consumption algorithm

\[
Income = x_i.m
\]
\[
C1 = 0.
\]

For $k$ from 1 to $\alpha$ do:

If $Income - b_{1k}$ ≥ 0 ∧ $x_i.g_1(t) < 100$

\[
x_i.y_{1k} = 1
\]

\[
Income = Income - b_{1k}
\]
\[
x_i.g_1(t) = x_i.g_1(t) + b_{1k}
\]
\[
x_i.g_2(t) = x_i.g_2(t) + b_{1k}
\]
\[
C1 = C1 + 1
\]

End consumption algorithm.

For non-alimentary goods:

Begin consumption algorithm

\[
Wealth = x_i.g_3(t - 1).
\]
\[
C2 = 0.
\]
For $k$ from $\alpha + 1$ to $r$ do:

If $\text{Wealth} - b_{1k} \geq 0$

\[ x_i \cdot y_{1k} = 1 \]

$\text{Wealth} = \text{Wealth} - b_{1k}$

\[ x_i \cdot g_1(t) = x_i \cdot g_1(t) + b_{1k} \]

\[ x_i \cdot g_2(t) = x_i \cdot g_2(t) + b_{1k} \]

$C2 = C2 + 1$

\[ x_i \cdot g_3(t) = x_i \cdot g_3(t) + \text{Wealth} \]

End consumption algorithm.

The main difference between these algorithms is that people consume alimentary goods based on their income restriction, but when they consume non-alimentary goods, they use monetary wealth in the immediately previous time period.

3.2. Increasing education levels

It is an action to generate dynamic model through income. An increment in the study will generate a raise in the individual's income. The instrumental variable related to education is $\beta_{0.1}(\alpha)$. The price of education is $\gamma(\alpha_{0.1})$. Education does not affect $\gamma_0$ or $\gamma_4$ directly. It is possible due to high cost of education, that individual should not always be able to consume it, therefore the algorithm should have provision option of knowing if it is consumed or not by a counter ($C3$).

Begin education algorithm

$C3 = 0$.

If $g_3(t) \geq b_{(r+1),1}$

\[ x_i \cdot y_{r+1}(t) = x_i \cdot y_{r+1}(t) + 1. \]

$C3 = 1$

\[ x_i \cdot g_3(t) = x_i \cdot g_3(t) - b_{(r+1),1} \]

\[ x_i \cdot m = (1 + \delta)x_i \cdot m \]

End education algorithm

3.3. The action of saving

In this model, the individual saves based on consuming alimentary goods of minor price exclusively. For that reason, the goods matrix ordered to upwardly in first column. Three alternatives for this action are presented by the model (See section 3.4) and a marginal propensity to saving, $s$.

The proposed algorithm for saving is the following:

Begin saving algorithm. Input: $s$

\[ \text{Income} = (1 - s)x_i \cdot m. \]

For $k$ from 1 to $\alpha$ do:

If $\text{Income} - b_{k1} \geq 0$

\[ x_i \cdot y_k = 1. \]

\[ \text{Income} = \text{Income} - b_{k1}. \]

\[ g_3(t) = g_3(t) + \text{Income}. \]

End saving algorithm.
Notice that there is no counter in the algorithm because a person must consume alimentary goods in order to survive. There is no permutation over the matrix good index since prices are sorted upwardly, and he consumes starting with the smallest price good as long as he can.

3.4. **Counters on the algorithms and extreme cases**

Under ideal conditions, you would execute always imitation actions if the individual's income is enough for acquiring the goods that others in their social circle have acquired. However, there is no motive that guarantees these actions are taken.

Some algorithms have a counter that allows knowing if these actions actually have made differences in a plan. The two first counters, $C_1$ and $C_2$ allow knowing how many goods have been consumed (alimentary and no alimentary respectively) for imitation process. The counter $C_3$ allows knowing if education has consumed or not in a given time (only you can take zero or one). There are three counters and eight possibilities.

<table>
<thead>
<tr>
<th>Case</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>Conditions and changing order in the actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>b</td>
<td>1</td>
<td>Best case scenario. Three actions take place and proceed checking the influence scheme.</td>
</tr>
<tr>
<td>2</td>
<td>a</td>
<td>b</td>
<td>0</td>
<td>Wealth is not enough to buy education. If $\Delta q_2 &gt; 0$ the order should be $A_1, A_3, A_2$</td>
</tr>
<tr>
<td>3</td>
<td>a</td>
<td>0</td>
<td>1</td>
<td>Alimentary goods and education has been consumed. In the next period the order should be $A_1, A_2, A_3$ if $q_{11} = -1$ and $A_2, A_1, A_3$ otherwise.</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>b</td>
<td>1</td>
<td>Case dismissed alimentary consumption goods must be first before education.</td>
</tr>
<tr>
<td>5</td>
<td>a</td>
<td>0</td>
<td>0</td>
<td>Wealth is also low to buy non-alimentary goods by imitation process. The next order should be $A_2, A_1, A_3$</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>b</td>
<td>0</td>
<td>Extreme case only when the order has been $A_2, A_1, A_3$. The individual has not consumed alimentary goods in this period of time Action $A_4$ must be taken immediately.</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>Case dismissed. The price of education must be higher than any other.</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Worst case scenario. Any imitation action has worked. Must execute action $A_4$ immediately.</td>
</tr>
</tbody>
</table>

Table 1. Possibilities over counters in the actions

3.5. **Steps for complete algorithm construction**

The steps for construction with all characteristics previously mentioned are:

1. Define initial conditions: Number of consumers, community of consumer (graph), income and initial consumption for each consumer, marginal propensity to save, rise by education, number of goods (both), goods matrix, permutations, and limit time of execution $T$.
2. Do the following cycle:
   - For $t$ from 1 to $T$ do:
     - For $i$ from 1 to $n$ do:
       - Execute actions based on the permutation order for consumer $x_i$.
       - Make modifications on permutation of consumer $x_i$ based on counter cases
       - And the influence scheme.

4. **Conclusions**

It is a model of bounded rationality that evidences consuming agents' behavior without processes of optimization, based in AAT. It contains simple and computationally economic rulers of decision, (fast and frugal), principal characteristic in theory of bounded rationality (Boyd and Richerson, 2002; Todd, 2002).
The advantage of consumption for imitation is when an individual consumption based on consumption of others instead of his/her consumption in a previous time, uncertainty on the contribution that goods makes to his goals is generated. Uncertainty also is a characteristic of bounded rationality (Simon, Edgi, Viale and Marris, 1992).

When calculating \( \sum_{l=1}^{n} x_l \cdot y_k (t) \), it represents number or consumers who have consumed good \( k \) for a \( t \) given, that put all goods together, a vector of demand of all goods to time is obtained, ideal for later studies.

In virtue of use, graphs theory (particularly no directed graphs) models a community of consumption (through the nodes) and relations among themselves (edges), a consumers' group like a society can be modeled who share information of consumption. This characteristic is desirable for those models that want to explain consequences that are generated for personal connections between the human beings.

The imitation learning process can complement with variables that have not been taken into account in the model, and they are described in experiments with human beings like described in Rodas (1974). Also a substantial variation in individuals' incomes can be included within the model, constructing social strata. According to Rodas, imitative processes can be influenced by various factors, like gender and social stratum, and so on. This model does not make distincion between men or women, which is a characteristic that can be added as parameter to each individual. Some others learning process can be used instead of imitative learning (Pingle, 1995; Sutton & Barto, 1998; Ghulam, 2010). Also imitative learning and reinforcement learning can combine to improve behavior in decision making modelling.

The model does not explain which ones are the initial values that should have the parameters. The choice of these values can be based on experimentation with real data or simulation processes.

As to initial values of instrumental variables, possibility of electing goods that have been a product of interaction in a particular community can be considered. It is here where social and cultural characteristics can be modeled. People who have grown in different places have different patterns of consumption, but when gathered in a single place, their habits mix to generate a different consumption behavior.

5. References


